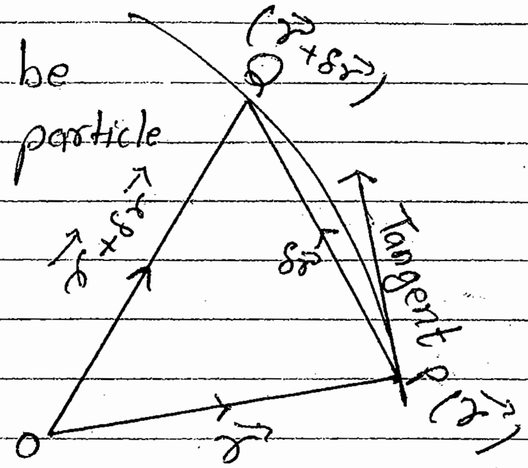
{VECTOR CALCULUS}

* Differentiation of Vectors :-



Let O be the origin and P be the position of a moving particle at time t.

Let $\vec{OP} = \vec{r}$

Let Q be the position of the particle at the time $t+\delta t$ and the position vector of

$$\vec{OQ} = \vec{r} + \delta\vec{r}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (\vec{r}+\delta\vec{r}) - \vec{r} = \delta\vec{r}$$

$\frac{\delta\vec{r}}{\partial t}$ is a vector. As $\delta t \to 0$, Q tends to P and the chord becomes the tangent at P.

We define.

$$\frac{d\vec{r}}{dt} = \lim_{\delta t \to 0} \frac{\delta\vec{r}}{\delta t}, \quad \text{then -}$$

$\frac{d\vec{r}}{dt}$ is a vector in the direction of the tangent at P.

$\frac{d\vec{r}}{dt}$ is called the differential coefficient of $\vec{r}$ with respect to 't'.

Similarly $\frac{d^2\vec{r}}{dt^2}$ is the second order derivative of $\vec{r}$.

$\frac{d\vec{r}}{dt}$ gives the velocity of the particle at P, which is along the tangent to its path. Also $\frac{d^2\vec{r}}{dt^2}$ gives the acceleration of the particle at P

$y = f(x) \rightarrow$ curve

$\frac{dy}{dx}$ = slope of tangent of the curve.

* **Vector Differentiation :-**

$\vec{r}$ is vector function of the variable t, then -

$$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{\frac{d\vec{r}}{dt} = \lim_{\delta t \to 0} \frac{\vec{r}(t+\delta t) - \vec{r}(t)}{\delta t}}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

Significance of derivative of vector function:-

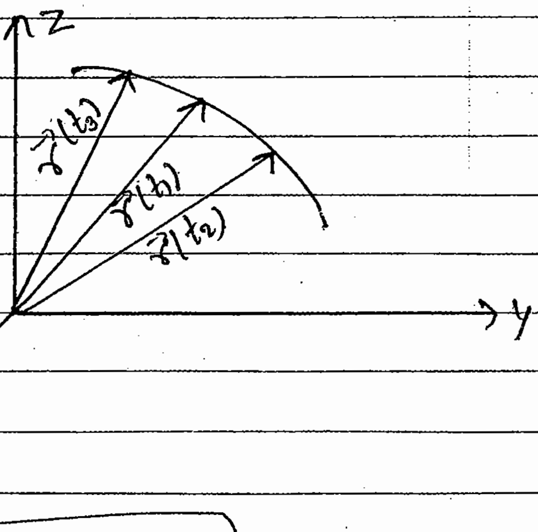
* $\vec{r}(t) \rightarrow$ displacement of the particle

$\frac{d\vec{r}(t)}{dt} \longrightarrow$ Velocity

$\frac{d^2\vec{r}(t)}{dt} \longrightarrow$ Acceleration.

* Tip of $\vec{r}(t)$ traces a curve.

$\frac{d\vec{r}}{dt}$ = tangent vector of the curve.



Unit tangent vector or direction of the Curve -

$$\boxed{\text{Unit Tangent Vector} = \frac{d\vec{r}/dt}{|d\vec{r}/dt|}}$$

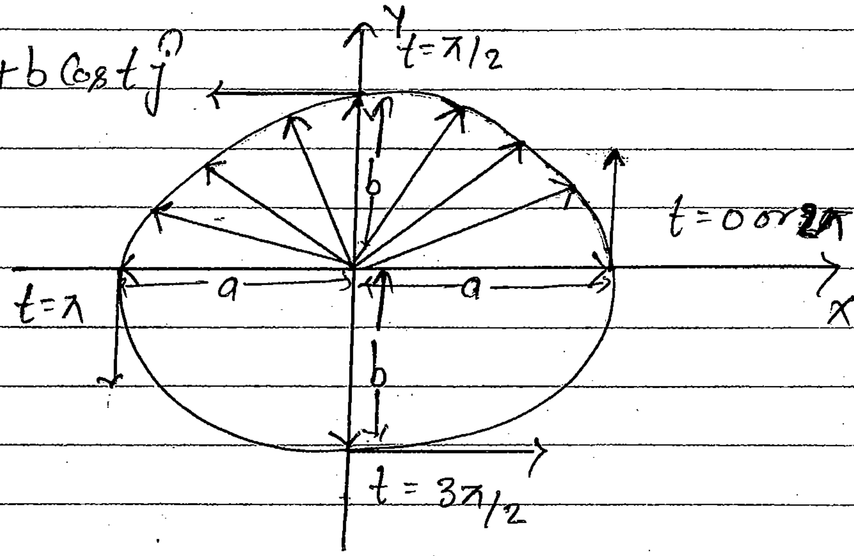
Anticlockwise or Counterclockwise is the positive direction of the curve.

* $\vec{r}(t) = a\cos t\,\hat{i} + b\sin t\,\hat{j}$

$\dfrac{d\vec{r}(t)}{dt} = -a\sin t\,\hat{i} + b\cos t\,\hat{j}$

at $t=0$ $= b\hat{j}$

at $t = \frac{\pi}{2}$ $= -a\hat{i}$



7-1

Q.20 A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. The components of its acceleration at $t=1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$ is equal to?

(a) $2/\sqrt{14}$ ✓(b) $-2/\sqrt{14}$ (c) $16/\sqrt{14}$ (d) $-16/\sqrt{14}$

Sol^n $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$

at $t=1$ direc^n $= \hat{i} - 3\hat{j} + 2\hat{k}$.

$\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$

$r(t) = (2t^2)\hat{i} + (t^2 - 4t)\hat{j} + (3t-5)\hat{k}$

$\dfrac{d\vec{r}(t)}{dt} = 4t\,\hat{i} + (2t-4)\hat{j} + 3\hat{k}$

$\dfrac{d^2 r(t)}{dt^2} = 4\hat{i} + 2\hat{j}$

$\left[\dfrac{d^2 r(t)}{dt^2}\right]_{t=1} \cdot \hat{A}$

$\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$(4\hat{i} + 2\hat{j})_{t=1} \cdot (\hat{i} - 3\hat{j} + 2\hat{k})$

$= 4\hat{i} - 6\hat{j}$